AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 1

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

The graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$ intersect at the points $(0, 0)$ and $(9, 3)$.

(a) $\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$

OR

$\int_0^3 \left(3y - y^2\right) dy = 4.5$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\pi \int_0^3 \left((3y+1)^2 - \left(y^2+1\right)^2 \right) dy$

$= \frac{207\pi}{5} = 130.061 \text{ or } 130.062$

$4 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

Form B
AB1
1A1

CALCULUS AB

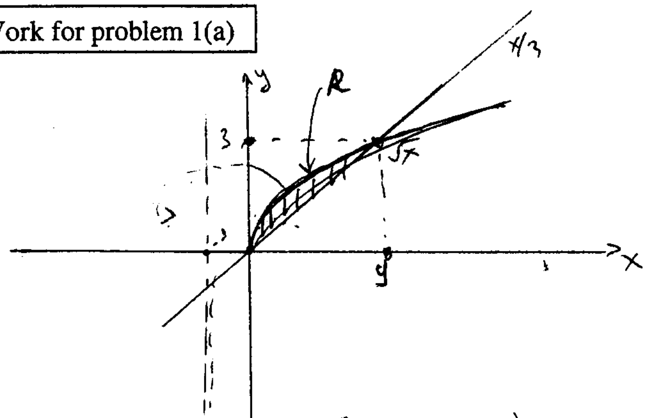
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)



$\frac{x}{3} = \sqrt{x}$

$\frac{x^2}{9} = x$

$9x = x^2$

$x = 9$ intersection coordinate

$R = \int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx \approx 4.5$ (by calculator)

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Continue problem 1 on page 5.

1A2

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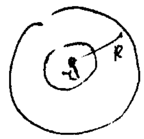
Work for problem 1(b)



$y = \frac{1}{3}x$

$x = 3y$

$y = \sqrt{x}$

$x = y^2$

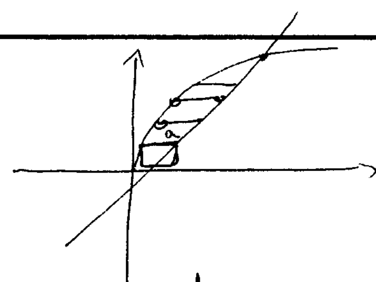
Use washer method:

R (radius) $= 3y + 1$

$r = y^2 + 1$.

$$\text{Volume} = \pi\int_0^3 (R^2 - r^2)\,dy = \pi\int_0^3 \left[(3y+1)^2 - (y^2+1)^2\right] dy = 130.0619 \quad \text{(by calc.)}$$

Work for problem 1(c)



If a cross section is square, then its area $A = (3y - y^2)^2$

$$\text{Volume} = \int_0^3 A\,dy = \int_0^3 (3y - y^2)^2\,dy = 8.1 \quad \text{(by calc.)}$$

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GO ON TO THE NEXT PAGE.

Form B

AB1

1B1

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

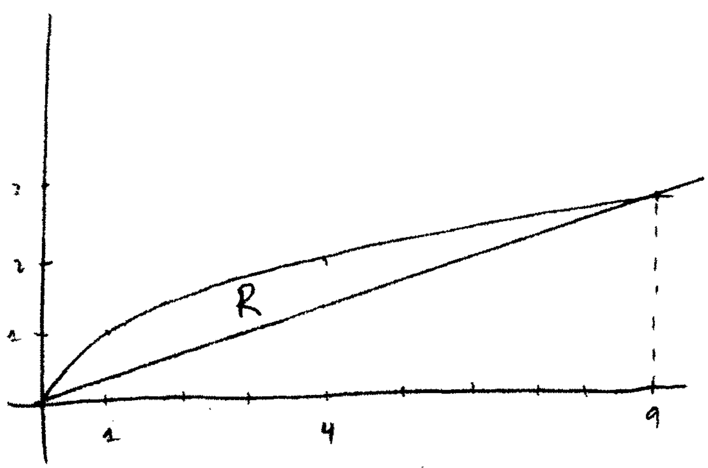
A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Intersects between $\sqrt{x}$ and $\frac{x}{3}$ at $x=9$ and $x=0$

$$R = \int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$$

the area is the area under the curve of $\sqrt{x}$ minus the area under the curve of $\frac{x}{3}$



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Continue problem 1 on page 5.

IB2

Work for problem 1(b)

R · big radius $R = 3y$
r · small radius $r = y^2$

$$V = \pi \int_0^3 (R)^2 - (r)^2 \, dy$$

$$V = \pi \int_0^3 (3y)^2 - (y^2)^2 \, dy = \frac{162}{5} = 32.4$$

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Work for problem 1(c)

$$V = \int_0^3 (3y - y^2)^2 \, dy = \frac{81}{10} = 8.1$$

GO ON TO THE NEXT PAGE.

Form B
AB1
1C1

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$y=\sqrt{x}$ $y=\frac{x}{3}$

$\sqrt{x}=\frac{x}{3}$ $x^{1/2}=\frac{x}{3}$ $x=9$

graphs intersects at $x=9$

First Quadrant

$$\int_0^9 \left(\sqrt{x}-\frac{x}{3}\right)dx = 4.5 \text{ units}^2$$

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Continue problem 1 on page 5.

1C$_2$

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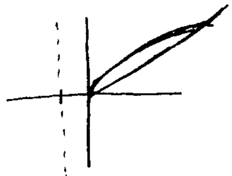
Work for problem 1(b)



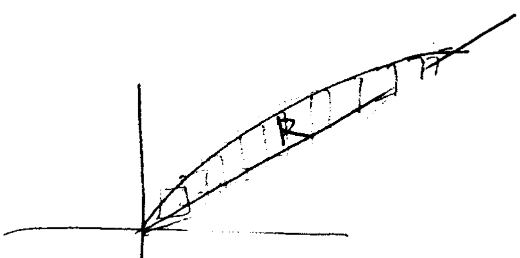
$y=\sqrt{x}$, $y=\frac{x}{3}$ $\qquad x=y^2 \qquad x=3y$

$dy=2y \qquad dy=3$

$\cancel{\pi\int_0^9 ((y^2)^2-(3y)^2)\,dy = 30230.9\ldots}$

$\pi\int_0^9 (1-y^2)^2-(1-3y)^2\,dy = 29467.5$ units

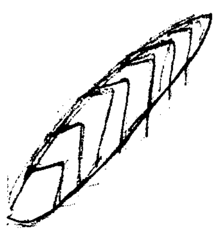
Work for problem 1(c)



area of R = 4.5 units^2

Volume = $\int_0^9 (\sqrt{x}-\frac{x}{3})^2\,dx$

= 2.7 units^2



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Question 1

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a), 1 point in part (b), and 2 points in part (c). The student presents correct work in parts (a) and (c). In part (b) the student has the correct limits and constant but rotates R about the line $x = 0$ instead of $x = -1$. As a result, the student earned only 1 point.

Sample: 1C
Score: 4

The student earned 4 points: 3 points in part (a), no points in part (b), and 1 point in part (c). The student presents correct work in part (a). In part (b) the student makes several errors. Although the constant is correct, the limits are incorrect, so the student did not earn the first point. The student attempts to rotate about $x = -1$ but has incorrect values in the integrand, so the response did not earn the integrand or answer points. In part (c) the student has the correct volume for cross sections drawn perpendicular to the <u>x-axis</u> and earned 1 point.

Question 2

For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

(a) $\int_0^2 r(t)\,dt = 206.370$ kilometers

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{dg}{dt} = \dfrac{dg}{dx} \cdot \dfrac{dx}{dt}; \quad \dfrac{dx}{dt} = r(t)$

$$\left.\frac{dg}{dt}\right|_{t=2} = \left.\frac{dg}{dx}\right|_{x=206.370} \cdot r(2)$$

$$= (0.050)(120) = 6 \text{ liters/hour}$$

$3 : \begin{cases} 2 : \text{uses chain rule} \\ 1 : \text{answer with units} \end{cases}$

(c) Let T be the time at which the car's speed reaches 80 kilometers per hour.

Then, $r(T) = 80$ or $T = 0.331453$ hours.

At time T, the car has gone

$$x(T) = \int_0^T r(t)\,dt = 10.794097 \text{ kilometers}$$

and has consumed $g(x(T)) = 0.537$ liters of gasoline.

$4 : \begin{cases} 1 : \text{equation } r(t) = 80 \\ 2 : \text{distance integral} \\ 1 : \text{answer} \end{cases}$

Form B
AB/BC 2
2A₁

Work for problem 2(a)

Distance travelled by car during first 2 hours $= \int_0^2 r(t)\,dt$

$= \int_0^2 120(1-e^{-10t^2})\,dt$

$= 206.370$ km

Work for problem 2(b)

$g(x) = 0.05x(1-e^{-x/2})$

Rate of change w.r.t. ~~at~~ time of $g(x) = \frac{dg(x)}{dt} = 0.05(1-e^{-x/2})\,r(t) + 0.05x(\frac{1}{2}e^{-x/2})\,r(t)$

∴ At time $t = 2$ hours, Rate of change w.r.t time of $g(x)$

$= 0.05(1-e^{(206.370/2)})\,r(2) + 0.05(206.370)(\frac{1}{2}e^{(-206.370/2)})\,r(2)$

$= 6$ L/hr.

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Continue problem 2 on page 7.

Work for problem 2(c)

$r(t) = 80$ km/hr $\Rightarrow 120(1 - e^{-10t^2}) = 80$

$\therefore t = 0.331$ hr

$\therefore$ Distance travelled $= \int_0^{0.331} r(t)\,dt$

$= \int_0^{0.331} 120(1 - e^{-10t^2})\,dt$

$= 10.794$ km

$\therefore$ Liters of gasoline used $= g(10.794)$

$= 0.05(10.794)(1 - e^{(10.794/2)})$

$= 0.537$ L

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Form B.
AB/BC 2
2B1

Work for problem 2(a)

$r(t) = 120(1-e^{-10t^2})$

$\int_0^2 r(t) = 206.37$

$\boxed{A: 206.37 \text{ km}}$

Work for problem 2(b)

$r(t) = \frac{dr}{dt} = 120(1-e^{-10t^2})$

$R(t) = \int 120(1-e^{-10t^2}) . \text{km}$

$g(R(t)) = 0.05x(1-e^{-x/2})$

$g'(R(t)) = 0.05e^{-\frac{x}{2}} . (e^{\frac{x}{2}} + 0.5(x-2))$

$\left\{ \begin{array}{l} \text{when } t = 2, \\ x = 206.37 \text{ km} \end{array} \right.$

$g'(x) = 0.05$

$\boxed{A: 0.05 \text{ liters per km}}$

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Continue problem 2 on page 7.

Work for problem 2(c)

$r(t) = 120(1-e^{-10t^2}) = 80$

$t = \cancel{-0.331453}$ or 0.331453

$R(t) = \int_0^{0.331453} r(t)$

$= 10.7941$ km

$\to g(10.7941) = 0.53726$

$\boxed{\text{A: } 0.537 \text{ liters}}$

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Form B
AB/BC 2
2C1

Work for problem 2(a)

the distance traveled $= \int_0^2 r(t)\,dt$

$= \int_0^2 120(1-e^{-10t^2})\,dt = 206.37$ km

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Work for problem 2(b)

$\dfrac{g'(x)}{r'(t)} =$

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Continue problem 2 on page 7.

Work for problem 2(c)

at the speed of 80

$$80 = 120(1 - e^{-10t^2})$$

we find (t) and subsated it in the result found in part ~~#~~ (b)

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Question 2

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), and 4 points in part (c). The student presents correct work in parts (a) and (c). In part (b) the student attempts to use the chain rule but does not put together the correct pieces necessary to answer the question.

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), and 1 point in part (c). The student presents correct work in part (a). No points were earned in part (b). In part (c) the student sets $r(t) = 80$ and earned the first point. Since the student does not solve the equation for t, the response did not earn the remaining points.

AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

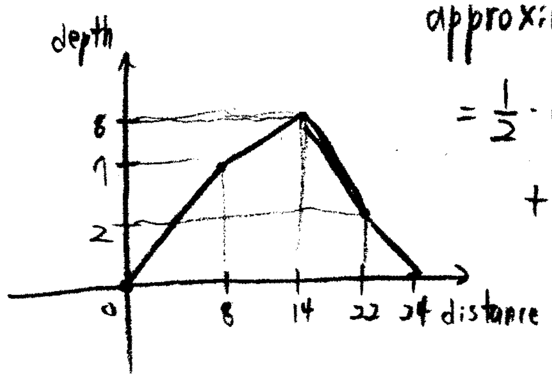
(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

Solution	Scoring
(a) $\frac{(0+7)}{2}\cdot 8 + \frac{(7+8)}{2}\cdot 6 + \frac{(8+2)}{2}\cdot 8 + \frac{(2+0)}{2}\cdot 2$ $= 115\text{ ft}^2$	1 : trapezoidal approximation
(b) $\frac{1}{120}\int_0^{120} 115v(t)\,dt$ $= 1807.169$ or $1807.170\ \text{ft}^3/\text{min}$	3 : { 1 : limits and average value constant; 1 : integrand; 1 : answer }
(c) $\int_0^{24} 8\sin\left(\frac{\pi x}{24}\right)dx = 122.230$ or 122.231 ft^2	2 : { 1 : integra1; 1 : answer }
(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is $\frac{1}{20}\int_{40}^{60} C\cdot v(t)\,dt = 2181.912$ or $2181.913\ \text{ft}^3/\text{min}$. Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds $2100\ \text{ft}^3/\text{min}$.	3 : { 1 : volumetric flow integral; 1 : average volumetric flow; 1 : answer with reason }

Form B
AB/BC 3
3A1

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

Work for problem 3(a)



approximation of area using trapezoidal sum

$$= \frac{1}{2}\cdot(8)\cdot 7 + \frac{1}{2}\underbrace{(7+8)}_{=15}\cdot\underbrace{(14-8)}_{=6}$$

$$+ \frac{1}{2}\cdot\underbrace{(8+2)}_{=10}\cdot\underbrace{(22-14)}_{=8} + \frac{1}{2}\cdot 2\cdot\underbrace{(24-22)}_{=2}$$

$$= 4\cdot 7 + 3\cdot 15 + 5\cdot 8 + 2$$

$$= 28 + 45 + 40 + 2$$

$$= 73 + 42 = 115$$

$115\ (ft)^2$

Work for problem 3(b)

Average value of volumetric flow at Picnic Point

$$= \frac{1}{120-0}\left(\int_0^{120} v(t)\,dt\right)\cdot 115 = \frac{115}{120}\int_0^{120}\left(16 + 2\sin\left(\sqrt{t+10}\right)\right)dt$$

$$= 1807.16491\ (ft)^3/\text{min}$$

Continue problem 3 on page 9.

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Work for problem 3(c)

$$\text{Area} = \int_0^{24} 8\sin\left(\frac{\pi x}{24}\right) dx$$

$\cos\pi = -1$

$\cos(0) = 1$

$$= \left[-\frac{24}{\pi}\cdot 8\cos\left(\frac{\pi x}{24}\right)\right]_0^{24} = -\frac{24\cdot 8}{\pi}\cos(\pi) + \frac{24\cdot 8}{\pi}\cos(0)$$

$$= -\frac{24\cdot 8}{\pi}\cdot(-1) + \frac{24\cdot 8}{\pi}\cdot(1)$$

$$= 2\cdot\frac{24\cdot 8}{\pi} = \underline{122.23099\,(ft)^2}$$

Work for problem 3(d)

Average value of volumetric flow during $40 \le t \le 60$

$$= \frac{1}{60-40}\left(\int_{40}^{60}\left(16 + 2\sin\left(\sqrt{t+10}\right)\right) dt\right)\cdot(122.23099)$$

$$= \frac{122.23099}{20}\cdot\int_{40}^{60}\left(16 + 2\sin\left(\sqrt{t+10}\right)\right) dt$$

$$= \underline{2181.91253\,(ft)^3/min} > 2100\,ft)^3/min$$

Thus, water must be diverted.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Form B
AB/BC 3
3B1

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

Work for problem 3(a)

24ft

Trapezoidal sum = $\dfrac{\text{Right sum} + \text{Left sum}}{2}$

Right sum $= (2\cdot 0) + (8\cdot 2) + (6\cdot 8) + (8\cdot 7)$
$= 120$

Left sum $= (8\cdot 0) + (6\cdot 7) + (8\cdot 8) + (2\cdot 2)$
$= 110$

Trapezoidal sum $= \dfrac{120+110}{2} = \boxed{115 \text{ square feet}}$

Work for problem 3(b)

Flow $= 115 \cdot v(t)$
$= 115 \cdot v(120)$
$= 115 \cdot 14.1627263406$
$= \boxed{1628.714 \text{ cubic feet per minute}}$

Continue problem 3 on page 9.

3B$_2$

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Work for problem 3(c)

(122.2309963)

$\int_0^{24} f(x)\,dx = \boxed{122.231 \text{ square feet}}$

Work for problem 3(d)

$\frac{1}{20}\int$ Volumetric flow > 2100 ft^3/min $\Rightarrow$ must be diverted.

$\frac{1}{60-40}\int_{40}^{60} (122.231)(V(t))\,dt$

$= \frac{1}{20}(43638.25299)$

$= \boxed{2181.913 \text{ ft}^3/\text{min}. \Rightarrow \text{Yes, this value indicates that the water must be diverted.}}$

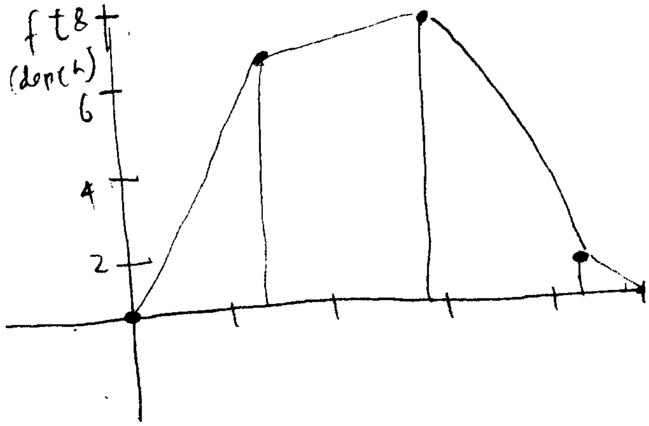
END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Form B
AB/BC 3
$3C_1$

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

Work for problem 3(a)



$\frac{1}{2}(b_1+b_2)h + \frac{1}{2}(b_2+b_3)h + \frac{1}{2}(b_3+b_4)h$

$+ \frac{1}{2}(b_4+b_5)h.$

$= \frac{1}{2}(0+7)8 + \frac{1}{2}(7+8)6$

$+ \frac{1}{2}(8+2)\times 8 + \frac{1}{2}(2+0)\times 2$

$= 115 \text{ ft}^2$

$\text{ft}^2 \times \text{ft/m}$

Work for problem 3(b)

$$\text{Avg} = \frac{\text{approx Area} \times V(0) + \text{approx Area} \times V(120)}{\cancel{120}}$$

$$= \frac{115 \times (16 + 2\sin(\sqrt{0+10})) + 115(16 + 2\sin(\sqrt{120+10}))}{\cancel{120}}$$

$= \cancel{28.9}$ ft³/minute

3463

Continue problem 3 on page 9.

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3C2

Work for problem 3(c)

$$\text{area} = \int_0^{24} f(x)$$

$$\int_0^{24} 8\sin\left(\frac{\pi}{24}x\right)dx = -8 \times \frac{24}{\pi} \times \cos\left(\frac{\pi}{24}x\right)\Big|_0^{24}$$

$$= 122.231\ ft^2$$

Work for problem 3(d)

$$\text{area} \times V(40) + \text{area} \times V(60)$$

$$= 122.231 \times (16 + 2\sin\sqrt{40+10}) + 122.231 \times (16 + 2\sin\sqrt{60+10})$$

$$= 4293.87\ ft^3/min$$

$\therefore$ must be diverted.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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Do not write beyond this border.

Question 3

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 1 point in part (a), no points in part (b), 2 points in part (c), and 3 points in part (d). The student presents correct work in parts (a), (c), and (d). In part (b) the student does not produce an integral, which is needed in order to find the average value of volumetric flow.

Sample: 3C
Score: 3

The student earned 3 points: 1 point in part (a), no points in part (b), 2 points in part (c), and no points in part (d). The student presents correct work in parts (a) and (c). In part (b) the student does not produce an integral to find the average value and thus did not earn any points. In part (d) the student also does not produce an integral and did not earn any points. Although the student's statement that the water "must be diverted" is true, the student does not present enough correct calculus work leading up to the answer to earn the answer point.

Question 4

The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4 + t^2}\, dt$ and $g(x) = f(\sin x)$.

(a) Find $f'(x)$ and $g'(x)$.

(b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

(c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

(a) $f'(x) = 3\sqrt{4 + (3x)^2}$

$g'(x) = f'(\sin x) \cdot \cos x$

$= 3\sqrt{4 + (3\sin x)^2} \cdot \cos x$

$4 : \begin{cases} 2 : f'(x) \\ 2 : g'(x) \end{cases}$

(b) $g(\pi) = 0$, $g'(\pi) = -6$
Tangent line: $y = -6(x - \pi)$

$2 : \begin{cases} 1 : g(\pi) \text{ or } g'(\pi) \\ 1 : \text{tangent line equation} \end{cases}$

(c) For $0 < x < \pi$, $g'(x) = 0$ only at $x = \frac{\pi}{2}$.

$g(0) = g(\pi) = 0$

$g\left(\frac{\pi}{2}\right) = \int_0^3 \sqrt{4 + t^2}\, dt > 0$

The maximum value of g on $[0, \pi]$ is
$\int_0^3 \sqrt{4 + t^2}\, dt$.

$3 : \begin{cases} 1 : \text{sets } g'(x) = 0 \\ 1 : \text{justifies maximum at } \frac{\pi}{2} \\ 1 : \text{integral expression for } g\left(\frac{\pi}{2}\right) \end{cases}$

Form B
AB 4
4A, 1

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$f'(x) = \frac{d}{dx}\left[\int_0^{3x} \sqrt{4+t^2}\, dt\right]$$
$$= 3\sqrt{4+9x^2}$$
$$g'(x) = \frac{d}{dx}\left[\int_0^{3\sin x} \sqrt{4+t^2}\, dt\right]$$
$$= 3\cos x \sqrt{4+9\sin^2 x}$$

Work for problem 4(b)

$$g'(\pi) = 3\cos\pi \sqrt{4+9\sin^2\pi}$$
$$= 3(-1)\sqrt{4+9(0)}$$
$$= -3(2)$$
$$= -6$$
$$g(\pi) = f(\sin\pi)$$
$$= f(0)$$
$$= 0$$

Equation of tangent line at $x = \pi$:

$$y = -6(x-\pi)$$

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Continue problem 4 on page 11.

Work for problem 4(c)

$g'(x) = 0$

$3\cos x \sqrt{4+9\sin^2 x} = 0$

$\Rightarrow 3\cos x = 0$

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2}$ is a critical point

Since g is continuous and differentiable on the interval $[0, \pi]$, by Extreme Value Theorem, the global maximum can occur at the critical points or end points

$g(\frac{\pi}{2}) = f(\sin\frac{\pi}{2})$
$= f(1)$
$= \int_0^3 \sqrt{4+t^2}\,dt > 0$

$\therefore$ The maximum value of g is $\int_0^3 \sqrt{4+t^2}\,dt$

$g(\pi) = f(\sin \pi)$
$= f(0)$
$= 0$

$g(0) = f(\sin 0)$
$= f(0)$
$= 0$

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GO ON TO THE NEXT PAGE.

Form B
AB4
4B1

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$f'(x) = \sqrt{4+(3x)^2} \cdot 3$$
$$= 3\sqrt{4+9x^2}$$
$$g'(x) = f'(\sin x) \cdot (\cos x)$$
$$= \cos x \cdot 3\sqrt{4+9\sin^2 x}$$
$$= 3\cos x\sqrt{4+9\sin^2 x}$$

Work for problem 4(b)

$$y = mx + b$$
$$m = g'(\pi) = 3\cos\pi\sqrt{4+9\sin^2\pi}$$
$$= -3\sqrt{4}$$
$$= -6$$
$$y = -6x + b$$
$$g(\pi) = f(\sin\pi) = f(0) = \int_0^0 \sqrt{4-t^2}\,dt = 0$$
$$0 = -6\pi + b$$
$$b = 6\pi$$
$$\boxed{y = -6x + 6\pi}$$

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Continue problem 4 on page 11.

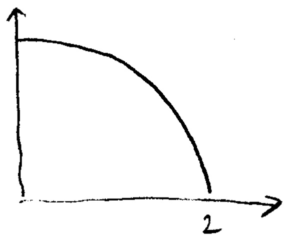
Work for problem 4(c)

$f(x) = \int_0^{3x} \sqrt{4+t^2}\, dt$

└ the formula for a circle with center (0,0) and radius of 2

Maximum value of g occurs when there is a maximum value of f.

f has its maximum value when $x = \frac{2}{3}$, because the answer will be the area of the quarter circle below.



This area is $= \int_0^2 \sqrt{4+t^2}\, dt$

$\therefore$ The maximum value is above

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Form B
AB4
4C1

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$f'(x) = \frac{d}{dx}\int_0^{3x} \sqrt{4+t^2}\,dt = 3\sqrt{4+(3x)^2}$.

$g'(x) = f'(\sin x) = 3\sqrt{4+(3\sin x)^2}$

Work for problem 4(b)

$g(\pi) = f(\sin\pi) = f(0) = 0$. $(\pi, 0)$

$g'(\pi) = f'(\sin\pi) = 3\sqrt{4+(0)^2} = 6$.

$y - 0 = 6(x - \pi)$

$y = 6x - 6\pi$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

$$\int_0^{\pi} 3\sqrt{4+(3\sin x)^2}\,dx$$

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Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

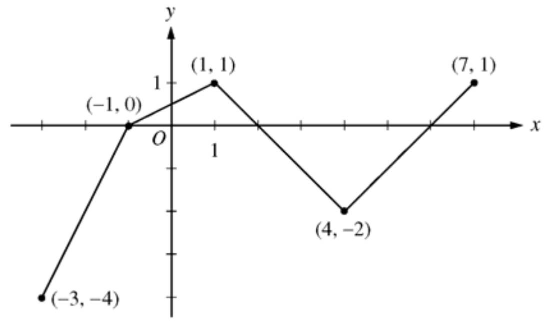
The student earned 6 points: 4 points in part (a), 2 points in part (b), and no points in part (c). The student presents correct work in parts (a) and (b). In part (c) the student tries to argue from a geometric point of view, but the initial premise is incorrect, so no points were earned.

Sample: 4C
Score: 4

The student earned 4 points: 2 points in part (a), 2 points in part (b), and no points in part (c). In part (a) the student has a correct $f'(x)$ but makes a chain rule error in finding $g'(x)$ so earned just 2 of the 4 points. In part (b) the student finds $g(\pi)$ correctly and finds a value for $g'(\pi)$ based on the incorrect answer in part (a). The student combines these values to form a tangent line equation, earning both points in part (b). The student's work in part (c) did not earn any points.

AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 5



Graph of g'

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

(a) g' changes from increasing to decreasing at $x = 1$;
g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x)\,dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$
$$g(2) = 5$$
$$g(7) = 5 + \int_2^{7} g'(x)\,dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \le x \le 7$ is $\frac{15}{2}$.

$3 : \begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$

(c) $$\frac{g(7) - g(-3)}{7 - (-3)} = \frac{\frac{3}{2} - \frac{15}{2}}{10} = -\frac{3}{5}$$

$2 : \begin{cases} 1 : \text{ difference quotient} \\ 1 : \text{ answer} \end{cases}$

(d) $$\frac{g'(7) - g'(-3)}{7 - (-3)} = \frac{1 - (-4)}{10} = \frac{1}{2}$$

No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

$2 : \begin{cases} 1 : \text{ average value of } g'(x) \\ 1 : \text{ answer "No" with reason} \end{cases}$

Form B AB/BC 5
5A1

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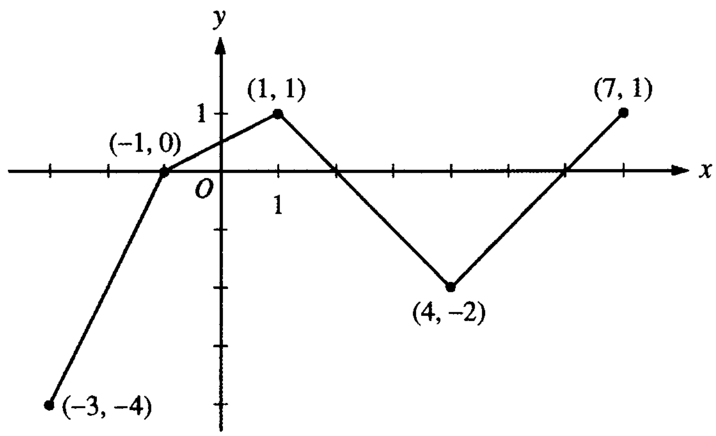

Graph of g'

Work for problem 5(a)

At points of inflection, g' should change from increasing to decreasing, or from decreasing to increasing.

Therefore, $x = 1, 4$

Work for problem 5(b)

$g'(x) = 0$ or the endpoints

$x=-3$: $\frac{7}{2} + 4 = \frac{15}{2}$

$x=-1$: $5 - 3 \times 1 \times \frac{1}{2} = \frac{7}{2}$

$x = 2$: $g(2) = 5$

$x = 6$: $g(6) = 5 - 4 \times 2 \times \frac{1}{2} = 1$

$x = 7$: $g(7) = 1 + 1 \times 1 \times \frac{1}{2} = \frac{3}{2}$

therefore, the absolute maximum is $\frac{15}{2}$ when $x = -3$.

Continue problem 5 on page 13.

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NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{g(7)-g(-3)}{7-(-3)} = \frac{\frac{3}{2}-\frac{15}{2}}{10} = \frac{\frac{-\cancel{12}^{6}}{\cancel{2}}}{10} = -\frac{3}{5}$$

Work for problem 5(d)

$$\frac{g'(7)-g'(-3)}{7-(-3)} = \frac{1-(-4)}{10} = \frac{1}{2}$$

But Mean Value Theorem doesn't guarantee a value of c such that $g''(c) = \frac{1}{2}$ because ~~function~~ function g' is not differentiable at some points.

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Form B AB/BC 5
5B₁

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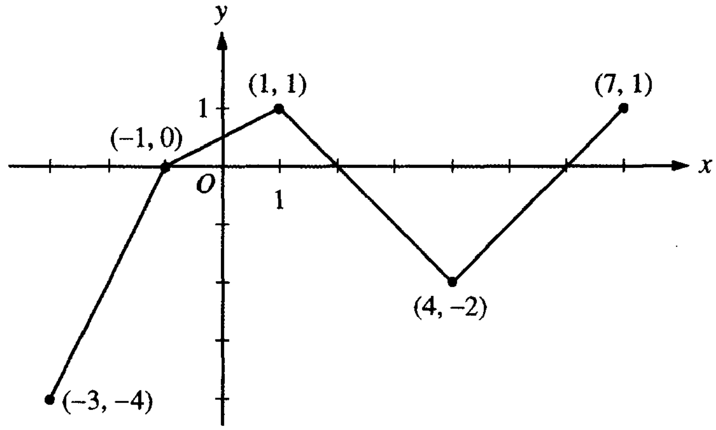

Graph of g'

Work for problem 5(a)

$\boxed{x=1,\ \ x=4}$

Since inflection of the graph ($g(x)$) is the point that $g'(x)$ increasing become decrease or $g'(x)$ decreasing become increase.

Work for problem 5(b)

$x=-3$

$g(-3)-4+\frac{3}{2}=5$

$\therefore g(-3)=\boxed{\frac{15}{2}}$

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Continue problem 5 on page 13.

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Work for problem 5(c)

$$\frac{g(7)-g(-3)}{7-(-3)} = \frac{\frac{3}{2}-\frac{15}{2}}{10} = \frac{-6}{10} = \boxed{-0.6.}$$

Meanwhile, $g(7)= g(2)-4+\frac{1}{2}$

$= 5-4+\frac{1}{2}$

$= \frac{3}{2}$ "

$g(-3)=\frac{15}{2}$ "

Work for problem 5(d)

$$\frac{g'(7)-g'(-3)}{7-(-3)} = \frac{1-(-4)}{10} = \boxed{\frac{1}{2}}$$

No, it doesn't guarantee. Since $g'(x)$ is not ~~derivati~~ derivated at $x=-1, 1, 4$.

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5 5 5 5 5

Form B AB/BC 5

5C1

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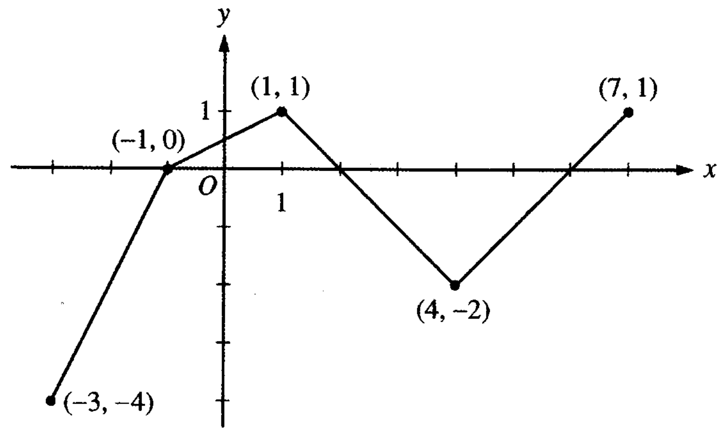

Graph of g'

Work for problem 5(a)

all points of inflection must have $f'(x)=0$ and change signs.

so $x = -1, 2$ and 6

Work for problem 5(b)

$\int_{-3}^{-1} g'(x)dx = 4$

$\int_{-1}^{2} g'(x)dx = 1.5$

$\int_{2}^{6} g'(x)dx = 4$

$\int_{6}^{7} g'(x)dx = 0.5$

→

since $g(2)=5$

so $g(-1) = 3.5$

$g(-3) = 7.5$

$g(6) = 1$

$g(7) = 1.5$

so the absolute maximum value of g on $-3 \le x \le 7$ is 7.5

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Continue problem 5 on page 13.

Work for problem 5(c)

average rate of change $= \dfrac{\int_{-3}^{7} g'(x)\,dx}{10}$

$= \dfrac{4+1.5+0.5+4}{10} = 1$

Work for problem 5(d)

average change of $g'(x) = \frac{2}{2} + \frac{\frac{1}{2}}{2} - \frac{1}{1} - \frac{1}{2} + \frac{1}{3} = \frac{1}{12}$

The Mean Value Theorem applied on the Interval $-3 \le x \le 7$ can not guarantee a value of c. for $-3 < c < 7$ such that $g''(c)$ is equal to this average change. Since the rate of change is not continuous on the interval

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). The student presents correct work in parts (a), (c), and (d). In part (b) the student does not identify $x = 2$ as a candidate, so the first point was not earned. The student finds the value of $g(-3)$ but does not find the value at the other endpoint, so the second point was not earned. The student did not earn the justification point since the work is not sufficient to state that $\frac{15}{2}$ is the maximum value.

Sample: 5C
Score: 4

The student earned 4 points: no points in part (a), 3 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student does not identify the correct values for the points of inflection. The student presents correct work in part (b). In part (c) the student uses the fact that the average rate of change is the average value of $g'(x)$ and presents a correct integral. The student makes an error in calculating the value of the integral so earned only 1 of the 2 points. In part (d) the student has an incorrect result for the average value of $g'(x)$, so the first point was not earned. Although the student declares that "[t]he Mean Value Theorem . . . can not guarantee a value of c" with the stated properties, the response includes the incorrect statement that "the rate of change is not continuous." Thus the student did not earn the second point.

AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 6

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

(a) $2x + 2 + 4y^3\dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 0$

$\left(4y^3 + 4\right)\dfrac{dy}{dx} = -2x - 2$

$\dfrac{dy}{dx} = \dfrac{-2(x+1)}{4\left(y^3+1\right)} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$

$2: \begin{cases} 1: \text{implicit differentiation} \\ 1: \text{verification} \end{cases}$

(b) $\left.\dfrac{dy}{dx}\right|_{(-2,\,1)} = \dfrac{-(-2+1)}{2(1+1)} = \dfrac{1}{4}$

Tangent line: $y = 1 + \dfrac{1}{4}(x+2)$

$2: \begin{cases} 1: \text{slope} \\ 1: \text{tangent line equation} \end{cases}$

(c) Vertical tangent lines occur at points on the curve where $y^3 + 1 = 0$ (or $y = -1$) and $x \neq -1$.

On the curve, $y = -1$ implies that $x^2 + 2x + 1 - 4 = 5$, so $x = -4$ or $x = 2$.

Vertical tangent lines occur at the points $(-4, -1)$ and $(2, -1)$.

$3: \begin{cases} 1: y = -1 \\ 1: \text{substitutes } y = -1 \text{ into the equation of the curve} \\ 1: \text{answer} \end{cases}$

(d) Horizontal tangents occur at points on the curve where $x = -1$ and $y \neq -1$.

The curve crosses the x-axis where $y = 0$.

$(-1)^2 + 2(-1) + 0^4 + 4 \cdot 0 \neq 5$

No, the curve cannot have a horizontal tangent where it crosses the x-axis.

$2: \begin{cases} 1: \text{works with } x = -1 \text{ or } y = 0 \\ 1: \text{answer with reason} \end{cases}$

Form B
AB 6
6 A_1

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Work for problem 6(a)

$2x + 2 + 4y^3 y' + 4y' = 0$

$y'(4y^3 + 4) = -2x - 2$

$y' = \dfrac{-2x-2}{4y^3+4} = \dfrac{-(x+1)}{2(y^3+1)}$

Work for problem 6(b)

$y' = \text{slope} = \dfrac{-(-2+1)}{2(1+1)} = \dfrac{1}{4}$

$y - y_0 = y'(x - x_0)$

$y - 1 = \frac{1}{4}(x+2)$

$y = \frac{1}{4}x + \frac{3}{2}$

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

line tangent to the curve is vertical $\Rightarrow$ slope is undefined

$2(y^3+1)=0$

$y^3+1=0$

$y^3=-1$

$y=-1$

$x^2+2x+1-4=5$

$x^2+2x-8=0$

$(x-2)(x+4)=0$

$x=2$ or $x=-4$ $\rightarrow$ The two points are $(2,-1)$ and $(-4,-1)$

Work for problem 6(d)

intersects the x-axis $\Rightarrow y=0$

$\frac{dy}{dx}=\frac{-(x+1)}{2}$ horizontal tangent is when slope $=0$

$\frac{-(x+1)}{2}=0$

$x+1=0$

$x=-1$

$+1-2+0+4(0)=5$

$-1\neq 5$ $\rightarrow$ No. It is not possible for this curve to have a horizontal tangent at points where it intersects the x-axis.

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Form B
AB6
6B1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$2x + 2 + 4y^3y' + 4y' = 0$$

$$4y^3y' + 4y' = -2x - 2$$

$$y'(4y^3 + 4) = -2x - 2$$

$$y' = \frac{-2x - 2}{4y^3 + 4}$$

$$y' = \frac{-\cancel{2}(x+1)}{2\cancel{4}(y^3+4)}$$

$$y' = \frac{-(x+1)}{2(y^3+1)} = \frac{dy}{dx}$$

Work for problem 6(b)

$$y' = \frac{-(-2+1)}{2(1^3+1)} \Rightarrow \frac{+1}{4}$$

$$y - y_0 = m(x - x_0)$$

$$y - 1 = \frac{1}{4}(x + 2)$$

$$y - 1 = \frac{1}{4}x + \frac{1}{2}$$

$$y = \frac{1}{4}x + \frac{3}{2}$$

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

Slope $= \infty$

$$y' = -\frac{(x+1)}{2(y^2+1)} = \infty$$

Work for problem 6(d)

horizontal pts $\Rightarrow y' = 0$

$\cancel{x}$ int - x-axis $\Rightarrow y = 0$

$$x^2 + 2x + y^4 + 4y = 5$$

$$x^2 + 2x - 5 = 0$$

$$\frac{-(x+1)}{2(y^2+1)} = 0 \Rightarrow -(x+1) = 0$$

$$-x - 1 = 0$$

$$-x = 1$$

$$x = -1$$

$$\Rightarrow (-1)^2 + 2(-1) - 5 = 0$$

$2 - 2 - 5 \neq 0$, its not possible.

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Form B AB6 6C1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$2x + 2 + 4y^3 \frac{dy}{dx} + 4\frac{dy}{dx} = 0$$

$$4y^3 \frac{dy}{dx} + 4\frac{dy}{dx} = -2x - 2$$

$$\frac{dy}{dx} (4y^3 + 4) = -2x - 2$$

$$\frac{dy}{dx} = \frac{-2x-2}{(4y^3+4)}$$

$$\frac{dy}{dx} = \frac{-2(x+1)}{4(y^3+1)} = \boxed{-\frac{(x+1)}{2(y^3+1)}}$$

Work for problem 6(b)

slope of tangent line $= \frac{dy}{dx}$ at $(-2, 1)$

$$\frac{dy}{dx} = \frac{-2(-2+1)}{2(1^3+1)} = \frac{4-2}{4} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x+2)$$

$$y = \frac{x}{2} + 1 + 1$$

$$\boxed{y = \frac{x}{2} + 2}$$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

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Work for problem 6(c)

$2(y^3+1) = 0$ means the virtical line

$2y^3 + 2 = 0$

$y^3 = -1$

$y = -1$

when $y = -1$ $x =$

Work for problem 6(d)

No, becaus it does not touch the x-axis

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AP® CALCULUS AB
2008 SCORING COMMENTARY (Form B)

Question 6

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). The student presents correct work in parts (a), (b), and (d). In part (c) the student does not present $y^3 = -1,$ so the response did not earn any points.

Sample: 6C
Score: 4

The student earned 4 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). The student presents correct work in part (a). In part (b) the student makes an error in calculating the slope so did not earn the first point. The student uses the incorrect slope and gives a tangent line equation, which earned the second point. In part (c) the student earned 1 point for finding $y = -1,$ but the response does not substitute the value of y in the original equation, so no additional points were earned. In part (d) the student presents an answer without any supporting work, so no points were earned.